Module Linear approximation using derivative

Linear approximation

1. Linear approximation

Using derivative, we can take a linear function which approximates the function nearby point of interest.

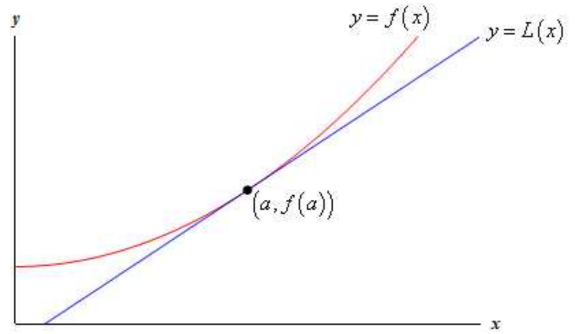
$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ (epsilon -> 0 as x -> a)

=> $\frac{f(x) - f(a)}{x - a} = f'(a) + \epsilon$ for x is near a

Epsilon is error, which is very small number

=> $f(x) = f(a) + f'(a)(x - a) + (x - a)\epsilon$

=> Let $L(x) = f(a) + f'(a)(x - a)$ and called linear approximation of f at x=a.



Using linear approximation, we can estimate the value $f(a+h)$.

$f(a+h) = L(a+h) + h\epsilon$ =f(a)+ f'(a) h + h(epsilon)

(We can not estimate epsilon right now. It is known that $\epsilon(h) \approx h$)

=> L(a+h) gives good estimation for f(a+h).

Example Estimate $\sqrt{4.05}$ using linear approximation.

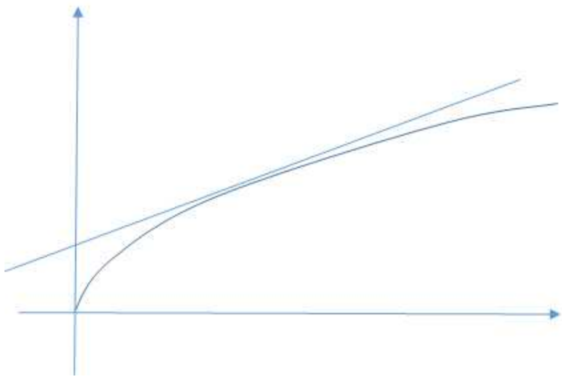
Use a function $f(x) = \sqrt{x}$ and f(4)=2 is known.

$f'(x) = \frac{d}{dx} x^{1/2} = \frac{1}{2} x^{-1/2}$ => $f'(4) = \frac{1}{2} \frac{1}{\sqrt{4}} = \frac{1}{4}$

=> Linear approximation of f at x=4 is

$$L(x) = f(4) + f'(4)(x - 4) = 2 + \frac{1}{4}(x - 4)$$

$\sqrt{4.05} = f(4+0.05) \approx L(4.05) = 2 + \frac{1}{4}(4.05-4) = 2 + \frac{1}{80} = 2.0125$



Remark) Your estimation should be given in rational numbers.

Exercise Estimate $\sqrt[3]{7.9}$ using linear approximation

Example) Portion of Korean population aged 65 and over is as follows

Year	Population percentage (%)
2013	12.2
2014	12.7
2015	13.1

Predict percentage at year 2017

Use linear approximation

P(t)=percentage of aged population at year t

=> want to estimate P(2017)

=> Use a linear approximation of P(t) at t=2014

$P(t) \approx P(2014) + P'(2014)(t-2014)$

Why do we take year 2014 as a reference year?

Average rate of change over 2013~2014 = 0.5

Average rate of change over 2014~2015 = 0.4

=> $P'(2014) \approx \frac{0.4+0.5}{2} = 0.45$

$P(2017) \approx P(2014) + P'(2014)(2017-2014) = 12.7 + (0.45) \times 3 = 14.05$

<= expected percentage at year 2017

(Real value at year 2017 : 13.8)