## Module Definite integral: definition

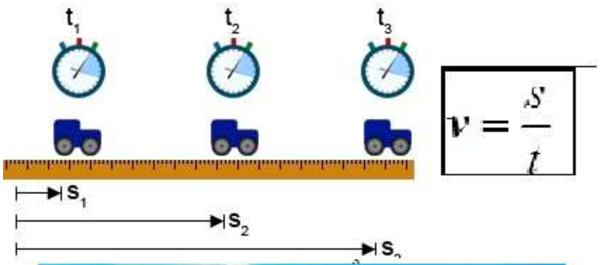
- Distance Problem
- Definite integral

### 1. Distance problem

Find the distance traveled by an object during a certain time period.

Velocity of the object is known at all times.

(You drive a car and keep watching speedometer of the car)




**Case)** Velocity is constant => distance = (velocity ) × (time)

**Case)** Velocity is not constant

Records of your speedometer

| Time (sec) | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|
| Velocity ( ft/sec) | 25 | 31 | 35 | 43 | 47 | 46 | 41 |

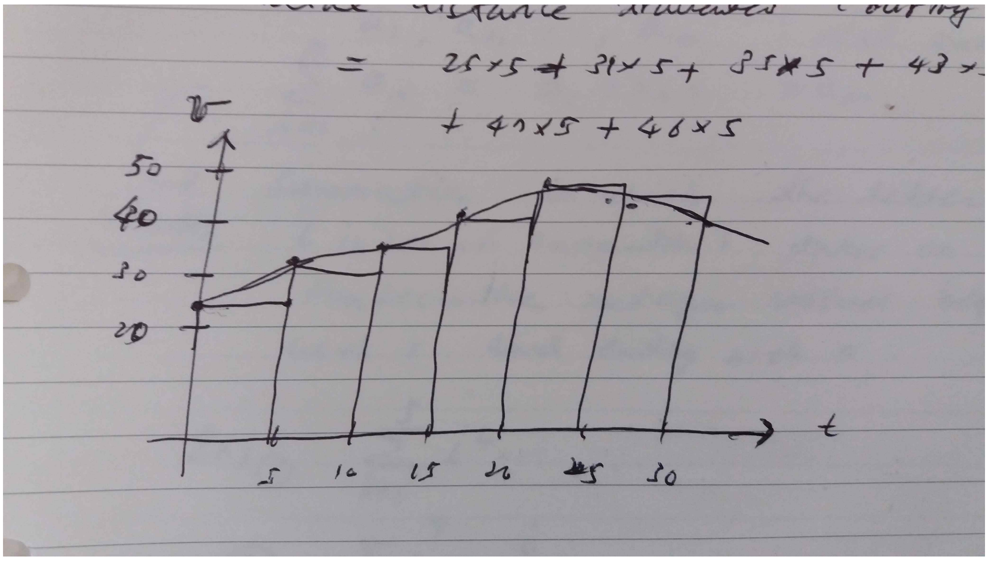
Approximately, traveled distance for first 5 seconds = $25 \times 5 = 125$ ft

Approximately, traveled distance for second 5 seconds = $31 \times 5 = 155$ ft

=>(Approximate) Total distance traveled during 30 seconds

= $25 \times 5 + 31 \times 5 + 35 \times 5 + 43 \times 5 + 47 \times 5 + 46 \times 5$

We can visualize total distance as follows




The sum of area of rectangles $\approx$ area below the curve (=graph of time-velocity function)
(=> actual velocity function takes changing values passing through the known points (time, velocity) )

=> $D = \lim_{n \to \infty} \sum_{j=1}^{n} v(t_j) \Delta t$

**Question)** The velocity of a moving object is given by $v = 10 - 4t \ \ (t \geq 0)$ meters.
What is the position of the object, that is displacement, after 5 seconds?
What is the total distance traveled by the object for 5 seconds?

## 2. Definite integral: Definition

A function f is defined over $a \leq x \leq b$.

Partition of [a, b ] =>

We divide the interval $[a,\ b]$ into n sub-intervals where each one has the length $(b-a)/n$. The j-th sub-interval is $[a+(j-1)(b-a)/n,\ a+j(b-a)/n]$. We define the righthand sum

$$R_n(f,[a,\ b]) := \sum_{j=1}^{n} f(a+j(b-a)/n)(b-a)/n$$

and the lefthand sum

$$L_n(f,[a,\ b]) := \sum_{j=1}^{n} f(a+(j-1)(b-a)/n)(b-a)/n$$

Definite integral of $f$ over $[a,\ b]$ is defined to be the following limit if it exists:

$$\lim_{n\to\infty} R_n(f,[a,\ b]) = \lim_{n\to\infty} L_n(f,[a,\ b])$$

Denote it with $\int_a^b f(x)dx$.